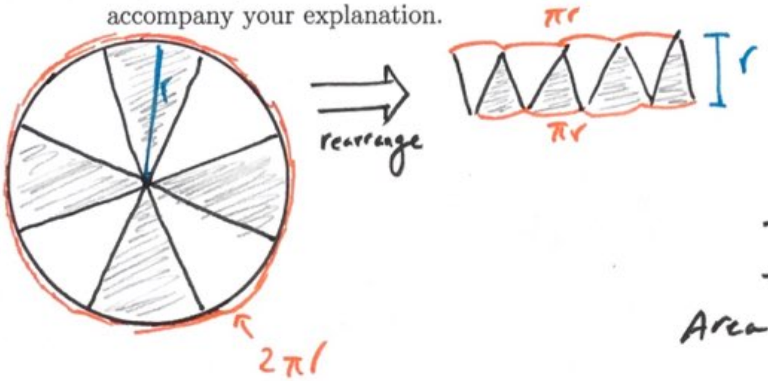
GMD-A1a

Practice Assessment Q4 #1

[NOTE: be ready to explain circumference, circle area, cylinder and/or cone surface area for real assessment]

1. Explain why the area of a circle with radius r can be found by $A=\pi * r^2$. You may use diagrams to accompany your explanation.



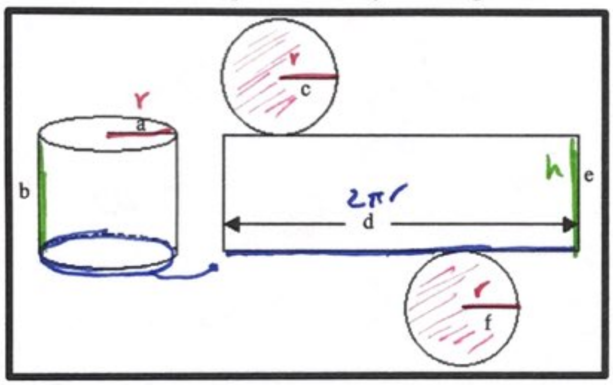
Rearrange slices of a circle into a 'rectangle'! The base of the rectangle is half the circumference: πr

The height of the 'rectangle' is the radius, r

Area = Base · Height = $\pi r \cdot r$

$= \pi r^2$ · QED

2. Label each part correctly. Then give the full formula for cylinder surface area.



a: radius, r b: height, h c: radius, r

d: Circumference $2\pi r$ e: height, h f: radius, r

SA $= 2 \cdot \pi r^2 + 2\pi r h$

GMD-A1b

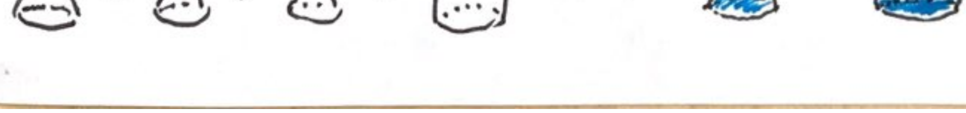
[NOTE: be ready to explain volume formulas for cylinders, cones, and prisms for this skill on real assessment.]

3. Explain why the volume of a cylinder can be found with $V = \pi * r^2 * h$. You may use diagrams to accompany your explanation.

A cylinder can be thought of as a stack of thin circles. Each circle has area πr^2. The total 'height' of the stack is a representation of how many circles there are, h.

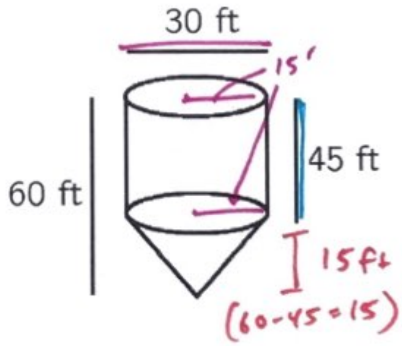
4. Explain why the volume of a cone with radius r and height h can be found by $V=\frac{1}{3} * \pi * r^2 * h$ You may use diagrams to accompany your explanation.

A cone with the same radius & height as a cylinder fills up only $\frac{1}{3}$ as much space as the cylinder. Since the cylinder has volume $\pi r^2 \cdot h$, the cone is $\frac{1}{3}\pi r^2 \cdot h$.



GMD-A2a

5. A silo is being used to store excess grain. It is shaped as shown, including a cone. The structure is 60 feet tall in total, 30 feet wide at its base, and the cylindrical portion is 45 feet tall. To the nearest whole number, find the volume of the silo. Include units in your answer.



Cylinder: $\pi r^2 \cdot h = \pi \cdot 15^2 \cdot 45 \approx 31{,}808.6$ ft³

Cone: $\frac{1}{3}\pi r^2 \cdot h = \frac{1}{3}\pi 15^2 \cdot 15 \approx 3534.3$ ft³

+

$\boxed{35{,}343 \text{ ft}^3}$

6. Find the surface area of the exposed portions of the silo pictured above to the nearest whole number. Include units in your answer.

Cone Surface Area:

SA: $\cancel{\pi r^2} + \pi r l$

only the $\pi r l$ part is exposed.

Cyl Surface Area:

SA: $2\pi r^2 + 2\pi r h$

only the rectangle and 1 circle are exposed.

$\pi r l + 1 \cdot \pi r^2 + 2\pi r \cdot h$

$15^2 + 15^2 = l^2$

$225 + 225 = l^2$

$450 = l^2 \Rightarrow l \approx 21.213$

SA: $\pi \cdot 15 \cdot 21.213 + \pi \cdot 15^2 + 2\pi(15)(45)$

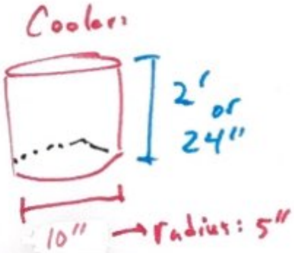
$= 999.639 + 706.858 + 4241.150$

$= \boxed{5948 \text{ ft}^2}$

MG-1a

7. Organizers are preparing water for an upcoming race. Each water cooler is a 2-foot tall cylinder with a 10-inch diameter. Water is distributed in cone-shaped cups 5 inches tall with a diameter of 4 inches. There are 300 runners and each runner is given 2 drinks. Approximately how many coolers are needed? Show the calculations that lead to your conclusion.

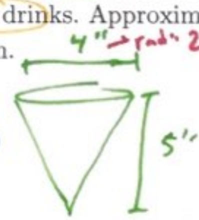
Coolers



$V_c = \pi r^2 \cdot h$

$= \pi \cdot (5^2) \cdot 24$

$V_c = 1885 \text{ in}^3$



$V_{cup} = \frac{1}{3}\pi r^2 \cdot h$

$V_{cup} = \frac{1}{3}\pi \cdot (2^2) \cdot 5$

$V_{cup} = 20.94 \text{ in}^3$

Need: 300 × 2 = 600 cups

Each cup is 20.94 in³

So we need 20.94 × 600 = 12564 in³ of water

12564 in³ needed
÷ 1885 in³ per cooler

6.66 coolers

$\boxed{\Rightarrow \text{Need } 7 \text{ coolers}}$

MG-A2a $V = \frac{4}{3}\pi r^3$ $\rightarrow r = \frac{2.02}{2}$ cm, $r = 1.01$ cm

8. A spherical rock with diameter 2.02cm is brought to your lab for identification. Its mass is measured on a scale to be 12.6g. Based on the table below, find the most likely category for the rock.

Type	Density (g/cm^3)
Shale	0.34
Graphite	2.23
Talc	2.92
Pyrite	5.02

2.02 cm

$r = 1.01$ cm

$V = \frac{4}{3}\pi (1.01)^3$

$V = 4.316$ cm^3

$$\text{Density} = \frac{\text{Stuff}}{\text{Space}} = \frac{\text{mass } (12.6g)}{\text{volume } (???)}$$

$$D = \frac{m}{V} = \frac{12.6g}{4.316cm^3} = 2.92\ g/cm^3$$

Talc!

9. Find the missing data values in the table. Round each to the nearest whole number.

Country	Total Population	Area (km^2)	Density (people/km^2)
Peru	29,555,000	1,285,000	23
Morocco	34,000,000	450,000	8
Laos	6,800,000	251,852	27

Density $= \frac{\text{stuff}}{\text{space}} = \frac{\text{ppl}}{\text{area}}$

Perú

$(1,285,000\ \cancel{km^2}) \times \left(23\ \frac{ppl}{\cancel{km^2}}\right)$

$= 29,555,000$

Morocco

$\frac{34,000,000\ ppl}{450,000\ km^2}$

$= 7.6$ ppl/km^2

≈ 8

LAOS

$D = \frac{ppl}{AREA}$

$(AREA)(D) = ppl$

$AREA = \frac{ppl}{D}$

$= \frac{6,800,000\ ppl}{27\ ppl/km^2}$

$= 251,852$ km^2

#1 (Alt. method)

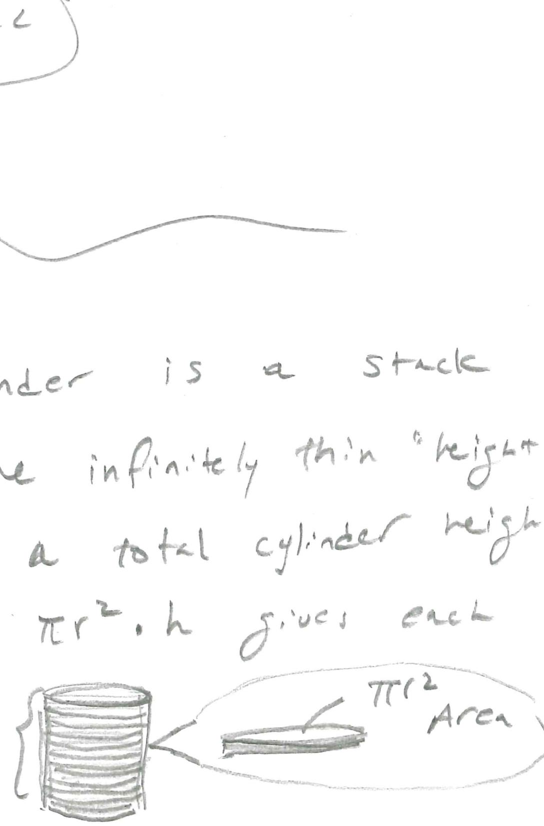
Subdivide circle into "rings". Longest/outer ring is the circumference, $2\pi r$.
"unravel" the rings into a triangle.

$$A = \frac{1}{2} \cdot b \cdot h$$
$$A = \frac{1}{2} \cdot 2\pi r \cdot r$$
$$A = \pi r^2$$

$$A = \frac{1}{2} b \cdot h$$
$$A = \frac{1}{\cancel{2}} \cdot \cancel{2}\pi r \cdot r$$
$$A = \pi r \cdot r$$
$$= \pi r^2$$

Bonus Facts:

- $\boxed{\text{Why is } Vol_{CYL} = \pi r^2 \cdot h \text{ ?}}$ A cylinder is a stack of circles. Each has area $= \pi \cdot r^2$. The infinitely thin "height" of each flat circle accumulates to give a total cylinder height equal to the height of the stack. So $\pi r^2 \cdot h$ gives each circle's size times the "number" of circles.



$\boxed{\text{Why is } C = 2\pi r\text{?}}$

π is defined as $\frac{\text{Circumference}}{\text{diameter}}$. So $\pi = \frac{C}{D}$. $D = 2r$ because 2 radii make a diameter. By substitution, $\pi = \frac{C}{2r}$. Multiply both sides by $2r$ to give $2\pi r = C$.