

Solutions

GMD-1a

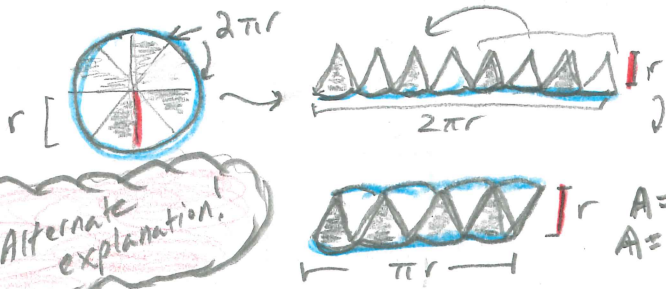
Practice Assessment

N.M.

See Notes Feb 2 or Feb 3

[NOTE: be prepared to explain circumference, circle area, cylinder and/or cone volume for real assessment]

1. Explain why the area of a circle with radius r can be found by $A = \pi \cdot r^2$. You may use diagrams to accompany your explanation.



A circle can be sliced into thin slices and rearranged into a "rectangle". More, thinner slices give a better rectangle. The base is half the circumference, πr ; the height is the radius, r . $A = B \cdot H$, or $A = \pi r \cdot r = \pi r^2$.

See pg 3 for Alternate explanation!

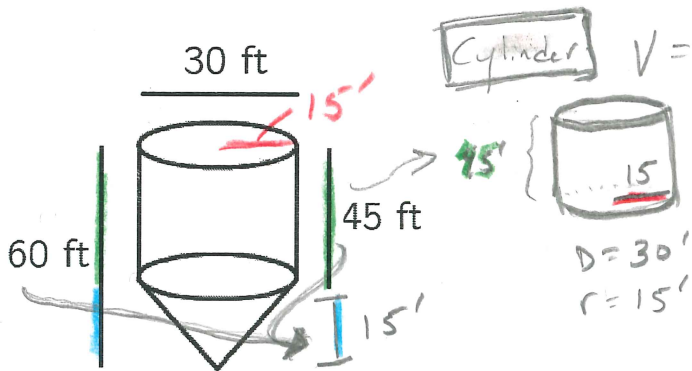
2. Explain why the volume of a cone with radius r and height h can be found by $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$. You may use diagrams to accompany your explanation.

A cylinder with height h and radius r has volume $= \pi r^2 \cdot h$.
A cone with the same radius and height occupies one-third the volume of the matching cylinder.



GMD-3a

3. A silo is being used to store excess grain. It is shaped as shown, including a cone. The structure is 60 feet tall in total, 30 feet wide at its base, and the cylindrical portion is 45 feet tall. To the nearest whole number, find the volume of the silo. Include units in your answer.



Cylinder

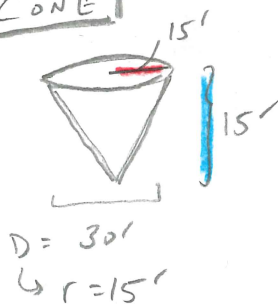
$$V = \pi r^2 \cdot h$$

$$V = \pi (15)^2 \cdot 45$$

$$= \pi \cdot 225 \cdot 45 = 10125\pi$$

$$\approx 31,808.6 \text{ ft}^3$$

CONE



$$V = \frac{1}{3} \pi r^2 \cdot h$$

$$= \frac{1}{3} \pi \cdot (15)^2 \cdot 15$$

$$= \frac{1}{3} \cdot \pi \cdot 225 \cdot 15$$

$$= 1125\pi$$

$$\approx 3534.3 \text{ ft}^3$$

total cylinder + cone =

$$35,351.9 \text{ ft}^3$$

$$\approx 35,352 \text{ ft}^3$$

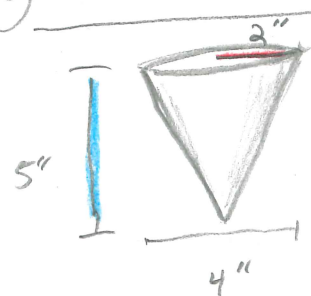
MG-1a

~~24~~ 24 inches!

4. Organizers are preparing water for an upcoming race. Each water cooler is a 2-foot tall cylinder with a 10-inch diameter. Water is distributed in cone-shaped cups 5 inches tall with a diameter of 4 inches. There are 300 runners and each runner is given 2 drinks. Approximately how many coolers are needed? Show the calculations that lead to your conclusion.

① How many cups? $300 \times 2 = \underline{600 \text{ cups}}$

② Water in each cup?



$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (2^2) \cdot 5$$

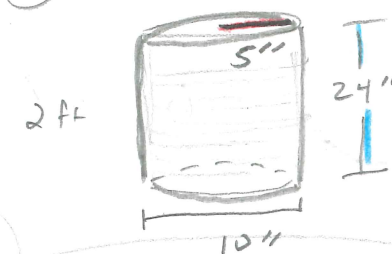
$$= \frac{1}{3} \pi \cdot 4 \cdot 5$$

$$\approx 20.94 \text{ in}^3$$

③ How much water needed?

$$20.94 \frac{\text{in}^3}{\text{cup}} \times 600 \text{ cups} = 12564 \text{ in}^3$$

④ Cooler volume?



$$V = \pi r^2 h$$

$$= \pi (5)^2 \cdot 24$$

$$= \pi \cdot 25 \cdot 24$$

$$= 600 \pi$$

$$\approx \underline{1885 \text{ in}^3}$$

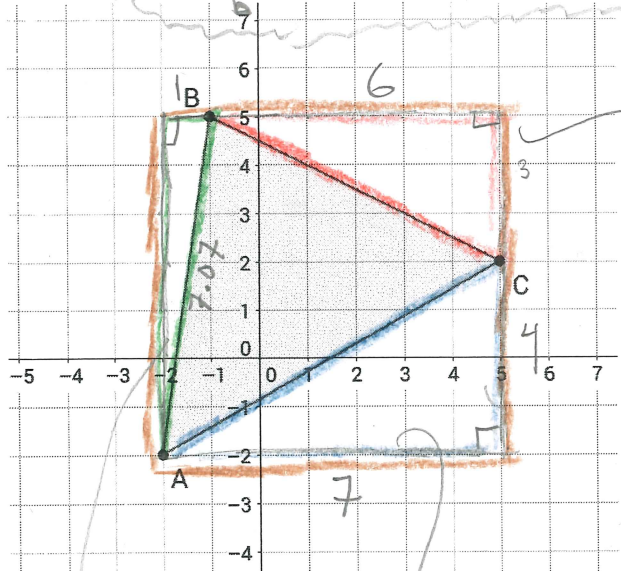
⑤ Dividing water into coolers:

$$\frac{12564 \text{ in}^3}{1885 \text{ in}^3/\text{cooler}} \approx 6.66 \text{ coolers}$$

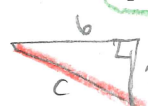
Need: 7 coolers
 w hew! Now I'm thirsty.

GPE-B2

$a^2 + b^2 = c^2$ Area = $\frac{1}{2} a \cdot b$



5. Find the perimeter of $\triangle ABC$ to the nearest tenth.



$$6^2 + 3^2 = c^2$$

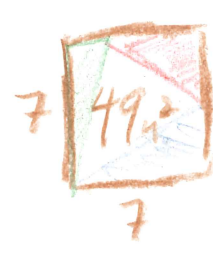
$$36 + 9 = c^2$$

$$45 = c^2$$

$$6.71 \approx c$$

$$7.07 + 6.71 + 8.06 = \underline{21.8}$$

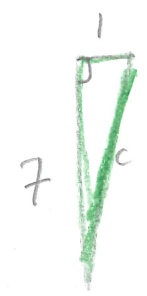
6. Find the area of $\triangle ABC$



$$A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 1 \cdot 7 = \underline{3.5}$$

$$A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 6 \cdot 3 = \underline{9}$$

$$A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 7 \cdot 4 = \underline{14}$$



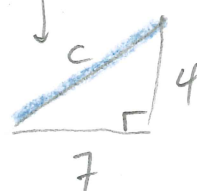
$$1^2 + 7^2 = c^2$$

$$1 + 49 = c^2$$

$$50 = c^2$$

$$\sqrt{50} = c$$

$$7.07 \approx c$$



$$7^2 + 4^2 = c^2$$

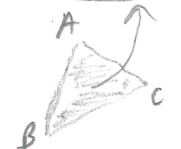
$$49 + 16 = c^2$$

$$65 = c^2$$

$$\sqrt{65} = c$$

$$8.06 \approx c$$

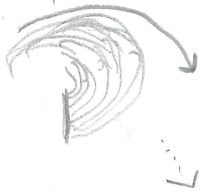
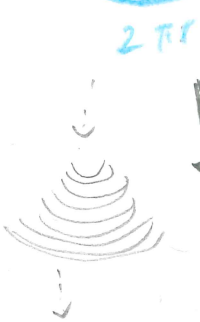
$$49 - 3.5 - 9 - 14 = \underline{22.5}$$



#1 (Alt. method)

Subdivide circle into "rings". Longest/outer ring is the circumference, $2\pi r$.

"unravel" the rings into a triangle.



$2\pi r$

$$A = \frac{1}{2} b \cdot h$$

$$A = \frac{1}{2} \cdot 2\pi r \cdot r$$

$$A = \pi r \cdot r = \pi r^2$$

$2\pi r$

$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot 2\pi r \cdot r$$

$$A = \pi r^2$$

Bonus Facts:

Why is $\text{Vol}_{\text{cyl}} = \pi r^2 \cdot h$? A cylinder is a stack of circles. Each has area = $\pi \cdot r^2$. The infinitely thin "height" of each flat circle accumulates to give a total cylinder height equal to the height of the stack. So $\pi r^2 \cdot h$ gives each circle's size times the "number" of circles.



Why is $C = 2\pi r$?

π is defined as $\frac{\text{Circumference}}{\text{diameter}}$. So $\pi = \frac{C}{D}$. $D = 2r$ because 2 radii make a diameter. By substitution, $\pi = \frac{C}{2r}$. Multiply both sides by $2r$ to give $2\pi r = C$.